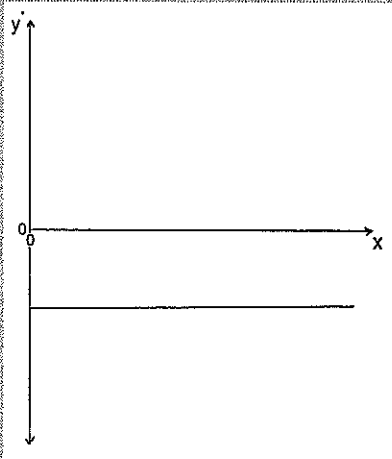


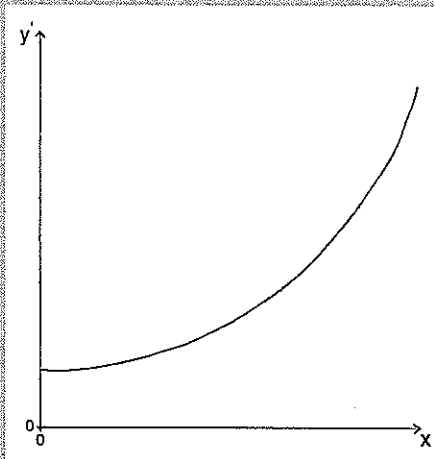
## Time Savers

Each group chooses only one graph to match statements and gradient function to.

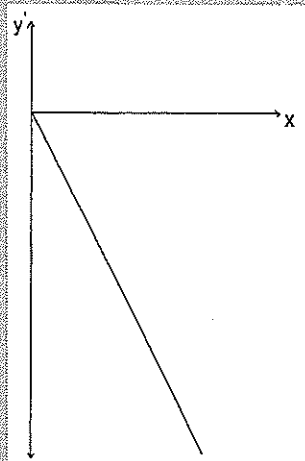
X.



Y.



Z.



### Problems:

1. True or False? Justify your choice.

- (i) Two different functions can have the same gradient function.
- (ii) An increasing function always has a positive gradient.
- (iii) A function that obeys  $y' \propto y$  must be an exponential function.
- (iv) A function which has a negative gradient for all values of  $x$  can only have negative values for  $y$ .
- (v) A function cannot have both negative and positive gradients.
- (vi) If  $y' > 0$ , then  $y'' > 0$ .

2. Explain in words how to differentiate the following:

- (i)  $y = ax^n$
- (ii)  $y = e^{ax}$

Describe an alternative method you could use to find the gradient at any point of a function.

*Measurement and Calculus level 8: ...choose and apply appropriate differentiation techniques in context.  
...identify situations where the rate of change is proportional to the value of the function and use the exponential function in associated problems.*



# Data Displays and Values

- Match four numbered statements to each of A,B and C.
- Match a, b and c to A,B and C. Justify your answers.
- Complete the task on page 16.
- Collect a set of numerical data from your class, eg. number of pens in pencil cases, length of foot in cm, etc. Draw a stem and leaf graph for the data. From this, draw a box and whisker graph of the data.

**A.**

177.6	1						
177.7	2	3					
177.8	1	3					
177.9	0	4	9	9			
178.0	1	1	2	3	5	6	
178.1	0	2	4	9	9		
178.2	0	0	0	1	3		

Lengths of surfboards (cm) DAY 1

1. The range is 0.67cm.

2. The mean is 177.9416cm.

3. The median length is 178.03cm.

4. The lower quartile of the data is 177.80cm.

5. The median result is 177.93cm.

6. The upper quartile is 178.00cm.

**B.**

177.6	0	4					
177.7	1	3	5				
177.8	2	2	7	9			
177.9	1	1	2	3	5	8	
178.0	2	4	4	4			
178.1	0	0	3	6			
178.2	1	7					

Lengths of surfboards (cm) DAY 2

7. This set of data has the smallest standard deviation.

8. The range of the results is 0.62cm.

9. The standard deviation for this data is 0.1747cm.

10. The modal value is 178.20cm.

11. There is no mode for this data.

**C.**

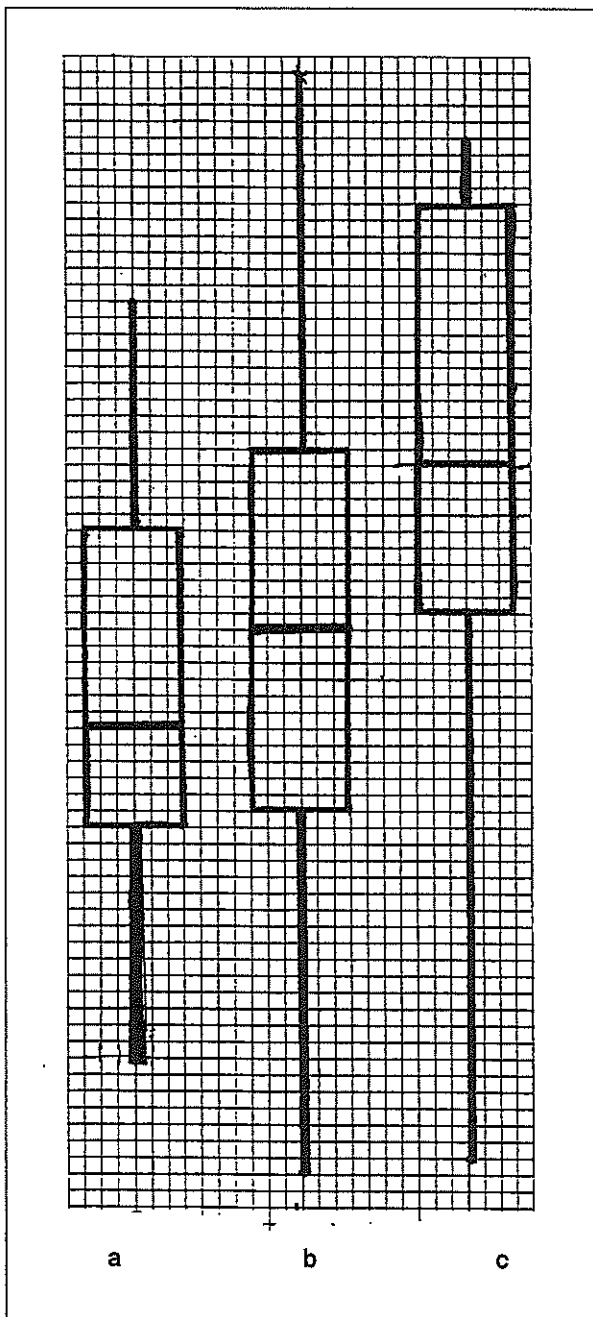
177.6	7	8	9				
177.7	5	6	9				
177.8	1	2	3	4	5	6	7
177.9	2	3	4	5	7	9	
178.0	0	4	6				
178.1	1	2	3				

Lengths of surfboards (cm) DAY 3

12. The upper quartile is 178.19cm.

## Time Savers

1. Each group finds the matching statements and graph for one of A,B or C.
2. Match A,B,C to a,b,c only.



### Task:

The data values used in the graphs in boxes A,B and C were recorded in quality control experiments over three consecutive days in a surfboard production factory.

The boards selected to be measured were the last twenty five to come off the production line on the previous day.

Your task is to write a brief report to the factory manager about the quality control results of these three days. You should include comments on;

- the consistency of the sample results over the three days.
- comments on the sampling method and any possible sources of bias.
- any recommendations for future sampling, and future production.



# Binomial, Normal and Poisson Distributions (1)

1. Find the two descriptor boxes (labelled a-f) that help describe each distribution.
2. Find the four numbered problems of each type, then solve them.
3. Make up one problem of each type for another group to solve. (Don't forget to work out the answers yourself so you will know whether or not the other group have answered them correctly!)
4. Discuss whether or not each of the numbered problems is modelled well by the distributions you chose. Decide whether all the conditions for that distribution are met.

A.

**Binomial Distribution**

a To use this distribution you must know the mean value and the standard deviation.

b This distribution models situations that have very small probability values.

c This distribution models situations which have two possible outcomes for each trial.

d This distribution is a model for some continuous random variables.

e This distribution models situations that have  $n$  independent trials and a constant probability of success.

f The distribution has only one parameter.

B.

**Normal Distribution**

C.

**Poisson Distribution**

1. The length of time the principal speaks in assembly is found to have a mean of seventeen minutes and a standard deviation of three and a half minutes. What is the probability she speaks for over 25 minutes?

2. Lengths of noses are normally distributed with a mean of 4.3 cm and a standard deviation of 0.15cm. What is the probability that a person chosen at random has an extremely dainty nose of 4cm or less?

3. Jeremiah's phone never seems to ring. On average he gets two phone calls a week. What is the probability that three weeks will go by with no calls?

4. Sandy arrives late for class on average four out of every ten days. What is the probability that Sandy is late for the next three consecutive days?

5. The mean number of times a student is asked a question in class is 1.3. What is the probability that Ben will not be asked a question today?

6. One third of the boys in Bright College play winter sport. If ten boys are chosen at random, find the probability that four or more play winter sport.

7. Jane loves chess. She wins 70% of the games she plays. What is the probability she will win most of her next ten games?

8. Steven does babysitting to earn some extra money. He usually has five babysitting sessions each month. If he earns twenty dollars a time, what is the probability he will earn over \$160 in the next two months?

## Time Savers

- Each group chooses one distribution to complete rather than completing all three.
- Each group matches all of a) -f) and 1-12 but solves only one of each type.

9. The average attention span of a seventeen year old is twenty two minutes, with a standard deviation of two and a half minutes. What is the probability that a randomly chosen seventeen year old has an attention span of more than 20 minutes?

10. Pat knew that she rarely passed a maths test. She calculated her probability of passing any maths test she sat was 0.1. It is her goal to pass three of the eight maths test this year. What is the probability she will pass three or more?

11. Weights of newly born New Zealand babies have a mean of 3.4 kg and a standard deviation of 0.25kg. What is the probability that the next baby born in New Zealand weighs between three and four kilos?

12. On average Robin cooks once every four days. What is the probability that Robin will cook twice in the next eight days?

**Statistics Level 8:...choose the appropriate distribution (binomial, Poisson or normal) to model a given situation, calculate probabilities and expected values, and make predictions using the model.**



## Binomial, Normal and Poisson Distributions (2)

1. Match four numbered statements with each of A,B & C.
2. a,b, and c each describe the conditions for one distribution to be used as an approximation for another. Match a, b and c to A,B & C.
3. x,y and z are examples of distribution questions that require using an approximation.

For each one (i) identify the original distribution (A,B or C), (ii) identify the appropriate approximation, and (iii) use the appropriate approximation to find the answer.

A.

**Poisson Distribution**

$$1. p(X = x) = \binom{n}{x} \pi^x (1 - \pi)^{n-x}$$

2. The probability of success for each trial must be constant.

3. For the Z distribution the mean is zero and the standard deviation is one.

4. There must be two possible outcomes for every trial.

5. The mean is  $np$ .

$$6. p(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

7. This distribution is for continuous random variables.

$$8. Z = \frac{x - \mu}{\sigma}$$

9. Events must not occur simultaneously.

10. Events must occur randomly.

11. This distribution is always symmetrical.

12.  $\lambda$  is the mean number of occurrences.

B.

**Normal Distribution**

C.

**Binomial Distribution**

**a**

This distribution can be modelled or approximated by the Poisson distribution when the probability of a trial resulting in success is either close to 0 or close to 1.

**x**

It has been found that 3% of "Greats Carrot Seed" do not germinate. If a keen gardener plants five hundred carrot seeds, what is the probability that fewer than ten of them will not germinate?

**b**

This distribution can be modelled by the normal distribution when the mean value is greater than 15.

**y**

Lawrence had never studied Japanese but decided to try the multichoice Japanese test his friends were sitting. He randomly chose answers to the 50 questions. Each question had three options, A, B or C. Find the probability that Lawrence answered 20 or more questions correctly.

**c**

This distribution is a good approximation for the binomial distribution when there is a large number of trials and when the probability of a trial resulting in success is near 0.5.

**z**

The number of phonecalls Suzie receives each week follows a Poisson distribution with a mean of 21.7. Find the probability that Suzie receives between 10 and 25 phonecalls next week.



# Sample and Population Statistics

- Match four numbered boxes to each of box A & B.
- Match the boxes a - l to the numbered boxes.  
A lettered box may be matched to more than one numbered box.  
Numbered boxes may be matched with more than one lettered box.
- In several sentences state the differences between sample and population statistics.

A.

**Sample Values**

B.

**Population Values**

- $\sigma$
- $\bar{x}$
- Census
- $\pi$
- s
- $\mu$
- p
- survey

a  $\frac{\sum x_i}{n}$  for ungrouped data

b  $\sqrt{\frac{\sum (x_i - \bar{x})^2 f_i}{\sum f_i}}$  for grouped data

c A farmer weighed 20% of her stock to find the mean weight of her animals.

d The sum of all the values divided by the total number of results.

e The number of people in New Zealand who smoke divided by the population of New Zealand.

f  $\sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$  for ungrouped data

g The number in the category of interest divided by the total number in the sample.

h This provides an estimate of the population proportion when the population value is not known.

i  $\frac{\sum x_i f_i}{\sum f_i}$  for grouped data.

j Twenty students from each form level were asked their opinions about developments for the new canteen.

k The Hutt City Council asked all residents of a particular street whether or not they thought it should be widened.

l In New Zealand this is done once every 5 years.





# Sequences and Series

## Time Savers

1. Match four numbered boxes to each of A-D. Decide whether each statement refers to a sequence or a series.
2. Find a real situation that could be modelled by each type of sequence or series. Write a question about it and swap with another group.

1. Find three matching statements for each of A-D.
2. Choose two of A-D. Find all four statements for each. Share your answers and reasons with the class.

A.

**Exponential**

B.

**Arithmetic**

C.

**Geometric**

D.

**Other**

1. Pip is laying a brick driveway. She decides to lay 20 bricks until it is complete.  
 $t_n$  = number of bricks laid on the nth day.

2. This type of sequence can be an increasing or a decreasing sequence.

3. This type of series always converges.

4.  $t_5 = 150$ ,  $t_7 = 275$ ,  $t_9 = 305$

5.  $a+ar+ar^2+ar^3+\dots$

6.  $\sum_{t=1}^n t$

7. This type of series does not converge.

8.  $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots$

9. This series converges when  $|r| < 1$

10.  $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$

11. A woman is about to have her fifth child. She predicts the length of her labour using the times for her previous labours; 10hrs, 8.5hrs, 6.3hrs and 4.9hrs.

12.  $a+(a+d) + (a+2d)+(a+3d)+\dots$

13. Peter decides to increase the amount of homework he does nightly by 10% each night.  
 $t_n$  = the amount of homework done on the nth night.

14.  $1+4x+\frac{16x^2}{2!}+\frac{64x^3}{3!}+\dots$

15. Azra finds that as she gets fitter her times to run 100 metres get quicker. Each run is done in 90% of the time of the previous run.  $t_n$  = length of time for the run on nth day.

16.  $\sum_{r=0}^n \binom{n}{r} a^{n-r} b^r$



## Information Sharing Activities

Each of the information sharing questions has three sets of clues. All the clues for question one should be combined to solve the problem.

Group members should exchange information by talking only, not by reading each other's clues.

There are 35 information sharing questions.

Maximising	q 1-3
Exponential Function	q 4-6
Uncertainties	q 7-8
Algorithms	q 9-11
Sequences and Series	q 12-14
Hyperbolas	q 15-16
Piecewise Functions	q 17-19
Binomial Theorem	q 20-21
Simultaneous Equations	q 22-23
Confidence Intervals	q 24-27
Probability Trees	q 28-30
Expected Value	q 31
Binomial Distribution	q 32-33
Poisson Distribution	q 34-35

Some empty sets of frames are on pages 32-33 for problems made up by you or by students, that can be kept for use in following years.

1.

Luana wants to maximise the volume of the boxes given that 90cm of ribbon is used on each one. What dimensions will give Luana the maximum volume?

3.

Find the gradient of the roller coaster for three different places, ie. for any three values of  $x$  within the domain.

1.

Luana enjoys making and packaging Turkish Delight for an exclusive sweet shop. The boxes used for the Turkish Delight have square ends.

3.

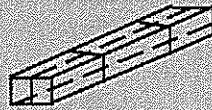
One section of a roller coaster follows the curve of the cubic function

$$y = \frac{-x^3}{3} + 6.5x^2 + 30x - 770$$

$$0 \leq x \leq 25$$

1.

Luana has a special way of decorating the box with ribbon, as shown in the diagram.



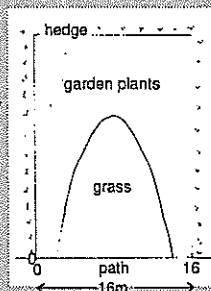
3.

Locate the turning point of the cubic function ie. the top of the rise in this section of the roller coaster.

2.

Toa draws a plan of the new garden on a numbered grid.

- 2 locate the exact position of the turning point.



4.

The rate of increase of the population of Expo city is proportional to the size of the population.

2.

Toa is planning a new section of garden. She wants to know the area of garden to be put in grass.

- 1 Find the positions where the curve meets the path and use integration to work out the area of the grassy section of the garden.

4.

In 1960 the population of Expo city was 120000.

Mathematically predict the population of the city in the year 2000.

2.

The equation of the curved border between the grassy area and garden plants is:

$$y = -x^2 + 16x - 28$$

4.

In 1990 the population of Expo city was 175003.

Write a function that relates the population and time.

5.

The concentration of a medical drug in the bloodstream decreases as time passes after the injection. The initial concentration is  $0.001 \text{ mg}/\ell$ . One hour later it is  $0.0006 \text{ mg}/\ell$ .

7.

As a group, calculate the volume of wood in a desk, or the volume of plastic in your ruler.

5.

The rate of change of the drug in the bloodstream is proportional to its concentration. When the concentration is below  $0.0001 \text{ mg}/\ell$  the drug is no longer effective.

7.

Estimate the uncertainties for all the measurements that are used in the calculation.

5.

Write the mathematical relationship between the concentration of the drug and time. Use it to find the time when the drug is no longer effective in the patient.

7.

Throughout the calculation calculate the cumulative effect of the uncertainties. Identify any stages of the calculation where errors are generated. Justify your decisions.

6.

Jerry finds that the rate of change of how interested he is in a history lesson is proportional to his interest at the time.

8.

Estimate the uncertainties for all the measurements that are used in the calculation.

6.

At what stage of the history lesson has Jerry only 40% of his initial interest in the lesson left?

8.

Throughout the calculation calculate the cumulative effect of the uncertainties. Identify any stages of the calculation where errors are generated. Justify your decisions.

6.

After 20 minutes of the first history lesson of the week, Jerry decides that he is only 60% as interested in the lesson as he was at the start.

8.

DO EITHER A OR B

- A. Calculate the volume of chalk in a full box of chalk sticks.
- B. Calculate the volume of aluminium used to make a drink can.

9.

371 is a magic number.

$$3^3 + 7^3 + 1^3 = 371$$

(i) Is 245 a magic number?

11.

Write an algorithm that will convert numbers to standard form.

9.

Write an algorithm that will find and output all the magic numbers less than five hundred.

11.

Use 200, 15, 0.09 and 3.5 in a desk check for your algorithm.

9.

Numbers are called magic numbers if they are equal to the sum of the cubes of their digits.

11.

Numbers in standard form have a decimal (with one digit to the left of the decimal point) multiplied by a power of 10.

eg.  $7.2 \times 10^2$ .

10.

A number is divisible by 5 if it ends in a 0 or a 5.

Use 9, 10, 12 and 15 in a desk check for your algorithm.

12.

Rita knows that her yo-yo returns to 85% of its previous height each time it goes up.

10.

A number is divisible by 3 if the sum of its digits is divisible by 3.

12.

Rita always starts the yo-yo from the top. The string is 30cm long.

10.

Construct an algorithm that will only output numbers that are divisible by both 3 and 5.

12.

How far will the yo-yo travel altogether if Rita lets it continue until it stops?

13.

George the dragon found that the flames from his mouth increased in length every time he roared.

15.

Sketch the function.

The function has a vertical asymptote at  $x = 6$ .

13.

George's first roar of the day had a flame an amazing 2.37m long.

- (i) How long will the flame be on the one thousandth roar of the day?

15.

Write the equation of the function.

The y-intercept of the function is  $(0, \frac{1}{2})$

13.

George's accountant measured several flames and found the increase was 2cm each time.

- (ii) How many roars ago was the flame 1.55m long?

15.

The function has a horizontal asymptote at  $y = 2$ .

The curve cuts the x-axis at  $x = \frac{-3}{2}$

14.

Steven made 3846 biscuits in the second month. If Steven continues the same pattern, how many biscuits will he have made in a year of baking?

16.

Production costs of a new maths book include initial costs for typesetting etc. (\$14,000) and \$5.80 for producing each copy.

14.

Steven had a compulsion to make biscuits. The more he made, the more he wanted to make!

In his first three months of biscuit making he had made 12,179 biscuits altogether!

16.

The publisher wants a profit of \$3.20 per book.

Sketch the function (# copies printed vs cost of a copy).

14.

In his first month, Steven had made 2564 biscuits.

After 12 months of this, Steven seeks help and manages to reduce his monthly output by 60% each month. When will he bake his last biscuit?

16.

The author's contract entitles them to six free copies of the book.

Write a function that relates  $y$ , (the cost of the book to buyers) to  $x$ , (the number of books printed).

17.

Sketch the function.

19.

Sene found that from the start of the party the amount of punch consumed was a linear function as time passed.

17.

The linear section has a gradient of 4 and goes through the point (2,7).

19.

Sene knew that for the time everyone went off for a midnight swim, no punch would be drunk but that when they returned the amount of punch consumed would follow a parabolic function until it was all gone.

17.

The domain is in two sections;  $x < -2$   
 $x \geq -2$

The parabolic section of the function has the equation  $y = (5 - x)(x + 1)$

19.

Sene was a rather antisocial partygoer. Instead of talking to people, he analysed how quickly the punch was being drunk. Sketch the function Sene might have found showing realistic values on the axes and stating the equations of the three sections of the piecewise function.

18.

There are three sections in the domain.

$$x < -1$$

$$-1 \leq x \leq 3$$

$$x > 3$$

20.

2 Use the general term of the binomial theorem to find the coefficient of the term in  $x^6$  of the series.

18.

Sketch the function. When  $x > 3$ ,  $y$  is constant with a value of 5.

20.

1 Use the binomial theorem to find the fourth term of the series.

18.

In the central section of the domain, the curve is hyperbolic with the function  $y = \frac{1}{x} + 2$

One section of the piecewise function has the equation  $y = \log(x + 5)$

20.

The series to be used in this question is  $(2 - 5x)^6$ .

21.

The series to use in this question are  $(14+2x)^5$  and  $(2+14x)^5$

23.

Joe, Louis and Sarah like to make their father work things out, so they tell him that between the three of them they have 15 socks.

21.

1 Find the first two terms for each series.

23.

Sarah, Joe and Louis are triplets. Their father wants to know how many socks each of them has to see whether or not to buy more.

Sarah tells her father that the number of socks she has plus twice the number Louis has is three more than Joe has.

21.

2 Which of the two series has a larger coefficient for the  $x^3$  term?

23.

Joe says he has one less sock than Sarah and Louis have between them. You take pity on their father and offer to work it all out for him.

How many socks do each of the triplets have?

22.

Solve the equations simultaneously  
 $x+2y+z = 6.9$

24.

Hemi uses the times he records over thirty school days to calculate the average driving time to school. The average time is 11.4 minutes.

22.

Solve the equations simultaneously  
 $x+y+z = 3.3$

24.

Hemi knows that the length of time taken for him to drive to school is a normal random variable with a standard deviation of 1.3 minutes.

22.

Solve the equations simultaneously  
 $2x+y-z = 8.4$

24.

Hemi leaves home earlier on Mondays to get to choir practice at 8am.

Find the 90% confidence interval for the mean length of time it takes Hemi to drive to school.



25.

Hair restoring cream is poured into bottles by a factory machine. Calculate the 90 and 95% confidence intervals for the mean volume of hair restorer expressed by the machine.

27.

It was found that 60% of the students sampled could roll their tongue. Discuss and agree on an appropriate method for finding a sample of 500 university students.

25.

The standard deviation of the amount of cream poured into the bottles is 3.7 mls. The manufacturer claims that every bottle contains at least 245 mls of hair restoring cream.

27.

Find the 96% confidence intervals for  
(a) the proportion of all university students who can roll their tongue.  
(b) the proportion of all university students who do not have attached ear lobes.

25.

A sample of 20 bottles is taken off the production line for a quality check. The volumes (mls) recorded were;  
240, 243, 243, 247, 257, 246, 245, 253, 251, 252, 241, 247, 242, 260, 263, 251, 250, 241, 260, 248.

27.

500 university students were surveyed on genetically linked characteristics. Only 23% of students surveyed had attached ear lobes.

26.

Stewart believes that Aucklanders are taller than people from the rest of the country. He sampled 50 relations of his in Auckland and 45 friends and relatives from around the country.

Do the results suggest Stewart is right about Aucklanders being taller?

28.

What is the expected number of keys Tracey needs to try in order to get the door unlocked?

26.

Stewart's results are:  
heights of Aucklanders  
 $\bar{x} = 1.65\text{m}$      $s = 0.08\text{m}$   
heights of other New Zealanders  
 $\bar{x} = 1.63\text{m}$      $s = 0.10\text{m}$   
Stewart is 1.75m tall.

28.

Tracey has four similar keys on her keyring. One of the keys will unlock her back door.  
  
She tries the keys in turn until the door opens.

26.

Discuss the accuracy of Stewart's sampling technique.

Calculate the 99% confidence intervals for the difference between the mean heights of Aucklanders and other New Zealanders.

28.

Being reasonably intelligent, Tracey does not try a key again if she finds it does not work.

Draw a probability tree to calculate the probabilities of the number of keys Tracey needs to try in order to open the door.

29.

The organiser of a national conference on cot death finds that 20% of the delegates are from overseas.  $\frac{2}{3}$  of the overseas delegates are women.

The organiser is pleased because there are more people at this conference than ever before.

31.

A weekly raffle of 100 tickets has five prizes; \$100, \$80, \$50, \$30 and \$10.

Tickets cost \$3.50.

29.

50% of the delegates to the conference are from the South Island, and  $\frac{1}{2}$  of them are women.

(ii) given that a randomly selected delegate is a woman, what is the probability she is from the South Island?

31.

If you buy a ticket every week, what will your average return be?

29.

Conference fees are \$210 per person. 30% of the delegates are from the North Island, and  $\frac{3}{4}$  of them are women.

(i) find the probability that a delegate chosen at random is a woman.

31.

Construct a table that shows all possible winnings and their probabilities. Are the tickets priced fairly? Justify your answer.

30.

He has coffee sixty percent of the time. On  $\frac{2}{3}$  of the days he has coffee, he has it with biscuits. He prefers chocolate biscuits.

32.

The probability that an overhead projector Mr Jones tries to use will not work is 0.45.

30.

When he drinks tea, he has cake  $\frac{3}{4}$  of the time.

If he is eating cake, what is the probability he is drinking coffee?

32.

Mr Jones seems to have little luck with overhead projectors. Some students think he should stick to chalk.

30.

An international executive always has difficulty deciding what to have for his morning tea. He has two choices to make; coffee or tea and cake or biscuits.

32.

In the next two weeks Mr Jones needs to use overhead projectors nine times. What is the probability that they will work for him at least five of the nine times?

33.

Varsha has found that she can do 85% of the maths problems she is faced with.

35.

Miriama and Damian washed 3 cars in the first half hour.

33.

Varsha's teacher usually expects students to try ten problems during a lesson.

35.

How many cars should they expect to wash in a two hour stretch if the initial rate continues?

What is the likelihood they wash at least this number?

33.

What is the probability that Varsha will be able to do nine or more problems on each of three consecutive days?

35.

In a school fundraising project Damian and Miriama are washing cars at a local shop.

34.

- 2 Find the probability that the shop sells
- (i) 4 in half an hour.
  - (ii) more than 4 in half an hour.

34.

A shop sells CDs at a constant rate of 10 each hour.

34.

- 1 Find the probability that the shop sells
- (i) only 15 in a 2 hr period.
  - (ii) less than 16 in a 2 hr period.

